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Interacting generalized Chaplygin gas model in non-flat universe

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Abstract. We employ the generalized Chaplygin gas of interacting dark energy to obtain the equation of state for the generalized Chaplygin gas energy density in a non-flat universe. By choosing a negative value for B we see that $w_\Lambda^{\text{eff}} < -1$, which corresponds to a universe dominated by phantom dark energy.

1 Introduction

One of the most important problems of cosmology is the problem of the so-called dark energy (DE). Type Ia supernova observations suggest that the universe is dominated by dark energy with negative pressure, which provides the dynamical mechanism of the accelerating expansion of the universe [1–3]. The strength of this acceleration is presently a matter of debate, mainly because it depends on the theoretical model implied when interpreting the data. Most of these models are based on the dynamics of scalar or multi-scalar fields. The primary scalar-field candidate for dark energy was the quintessence scenario [4–7], a fluid with the parameter of the equation of state lying in the range, $-1 < w < \frac{-1}{3}$. While the most model independent analysis suggests the acceleration of the universe to be below the de Sitter value [8–11], it is certainly true that the body of observational data allows for a wide parameter space, compatible with an acceleration larger than the de Sitter one [12–14]. If eventually this proves to be the case, the fluid driving the expansion would violate not only the strong energy condition $\rho + 3P > 0$, but the dominant energy condition $\rho + P > 0$ as well. Fluids of such characteristic are dubbed a phantom fluid [15]. In spite of the fact that the field theory of phantom fields encounters the problem of stability, which one could try to bypass by assuming them to be effective fields [16–25], it is nevertheless interesting to study their cosmological implications. Recently there have appeared many relevant studies on phantom energy [26–40]. The analysis of the properties of dark energy from recent observations mildly favors models with w crossing -1 in the near past. So far, a large class of scalar-field dark energy models have been studied, including quintessence, K-essence [41, 42], tachyon [43, 44], ghost condensate [45, 46] and quintom models [47–53], and so forth. In addition, other proposals on dark energy include interacting dark energy models [54–60], braneworld models [61–63], and holographic dark energy models [64–76], etc.

In a very interesting paper Kamenshchik, Moschella and Pasquier [77] have studied a homogeneous model based on a single fluid obeying the Chaplygin gas equation of state

$$P = \frac{-A}{\rho}\,, \tag{1}$$

where P and ρ are respectively pressure and energy density in a comoving reference frame, with $\rho > 0$; A is a positive constant. This equation of state has attained a certain interest [78–82] because of its many interesting and, in some sense, intriguingly unique features. Some possible motivations for this model from the field theory point of view are investigated in [83, 84]. The Chaplygin gas emerges as an effective fluid associated with d-branes [85, 86] and can also be obtained from the Born–Infeld action [87, 88].

Inserting the equation of state (1) into the relativistic energy conservation equation leads to a density evolving as

$$\rho_\Lambda = \sqrt{A + \frac{B}{a^6}}\,, \tag{2}$$

where B is an integration constant.

In the present paper, using the generalized Chaplygin gas model of dark energy, we obtain the equation of state for an interacting Chaplygin gas energy density in a non-flat universe. The currently available observational data imply that the dark energy behaves as a phantom-type dark energy, i.e. the equation of state of dark energy crosses the cosmological-constant boundary $w = -1$ during the evolution. We show this phantomic description of the interacting generalized Chaplygin gas dark energy in a non-flat universe with $B < 0$.

2 Interacting generalized Chaplygin gas

In this section we obtain the equation of state for the generalized Chaplygin gas when there is an interaction between the generalized Chaplygin gas energy density ρ_Λ and

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the cold dark matter (CDM), with $w_{\rm m}=0$. The continuity equations for dark energy and CDM are

$$\dot\rho_\Lambda+3H(1+w_\Lambda)\rho_\Lambda=-Q, \quad (3)$$
$$\dot\rho_{\rm m}+3H\rho_{\rm m}=Q\,. \quad (4)$$

The interaction is given by the quantity $Q=\Gamma\rho_\Lambda$. This corresponds to the decay of the generalized Chaplygin gas component into CDM with the decay rate Γ. Taking the ratio of the two energy densities as $r=\rho_{\rm m}/\rho_\Lambda$, the above equations lead to

$$\dot r=3Hr\left[w_\Lambda+\frac{1+r}{r}\frac{\Gamma}{3H}\right]. \quad (5)$$

Following [89], if we define

$$w_\Lambda^{\rm eff}=w_\Lambda+\frac{\Gamma}{3H}\,,\quad w_{\rm m}^{\rm eff}=-\frac{1}{r}\frac{\Gamma}{3H}\,, \quad (6)$$

then the continuity equations can be written in their standard form,

$$\dot\rho_\Lambda+3H\left(1+w_\Lambda^{\rm eff}\right)\rho_\Lambda=0\,, \quad (7)$$
$$\dot\rho_{\rm m}+3H\left(1+w_{\rm m}^{\rm eff}\right)\rho_{\rm m}=0\,. \quad (8)$$

We consider the non-flat Friedmann–Robertson–Walker universe, with line element

$$ds^2=-dt^2+a^2(t)\left(\frac{dr^2}{1-kr^2}+r^2d\Omega^2\right), \quad (9)$$

where k denotes the curvature of space $k=0,1,-1$ for a flat, closed and open universe, respectively. A closed universe with a small positive curvature ($\Omega_k\sim0.01$) is compatible with the observations [90–92]. We use the Friedmann equation to relate the curvature of the universe to the energy density. The first Friedmann equation is given by

$$H^2+\frac{k}{a^2}=\frac{1}{3M_p^2}\left[\rho_\Lambda+\rho_{\rm m}\right]. \quad (10)$$

Let us define as usual

$$\Omega_{\rm m}=\frac{\rho_{\rm m}}{\rho_{\rm cr}}=\frac{\rho_{\rm m}}{3M_p^2H^2},\quad \Omega_\Lambda=\frac{\rho_\Lambda}{\rho_{\rm cr}}=\frac{\rho_\Lambda}{3M_p^2H^2},$$
$$\Omega_k=\frac{k}{a^2H^2}\,. \quad (11)$$

Now we can rewrite the first Friedmann equation as

$$\Omega_{\rm m}+\Omega_\Lambda=1+\Omega_k\,. \quad (12)$$

Using (11) and (12) we obtain the following relation for the ratio of energy densities r:

$$r=\frac{1+\Omega_k-\Omega_\Lambda}{\Omega_\Lambda}\,. \quad (13)$$

In the generalized Chaplygin gas approach [87, 88], the equation of state (1) is generalized to

$$P_\Lambda=\frac{-A}{\rho_\Lambda^\alpha}\,. \quad (14)$$

The above equation of state leads to a density evolution

$$\rho_\Lambda=\left[A+\frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}. \quad (15)$$

Taking the derivative in both sides of the above equation with respect to the cosmic time, we obtain

$$\dot{\rho_\Lambda}=-3BHa^{-3(1+\alpha)}[A+Ba^{-3(1+\alpha)}]^{\frac{-\alpha}{1+\alpha}}\,. \quad (16)$$

Substituting this relation into (3) and using the definition $Q=\Gamma\rho_\Lambda$, we obtain

$$w_\Lambda=\frac{B}{a^{3(1+\alpha)}\left[A+Ba^{-3(1+\alpha)}\right]}-\frac{\Gamma}{3H}-1\,. \quad (17)$$

Here as in [93], we choose the following relation for the decay rate:

$$\Gamma=3b^2(1+r)H\,, \quad (18)$$

with the coupling constant b^2. Using (13), the above decay rate takes the following form:

$$\Gamma=3b^2H\frac{(1+\Omega_k)}{\Omega_\Lambda}\,. \quad (19)$$

Substituting this relation into (17), one finds the generalized Chaplygin gas energy equation of state

$$w_\Lambda=\frac{B}{a^{3(1+\alpha)}[A+Ba^{-3(1+\alpha)}]}-\frac{b^2(1+\Omega_k)}{\Omega_\Lambda}-1\,. \quad (20)$$

Now, using the definition generalized Chaplygin gas energy density ρ_Λ, and using Ω_Λ, we can rewrite the above equation as

$$w_\Lambda=\frac{B}{(3M_p^2H^2a^3\Omega_\Lambda)^{1+\alpha}}-\frac{b^2(1+\Omega_k)}{\Omega_\Lambda}-1\,. \quad (21)$$

From (6), (19) and (21), we have the effective equation of state

$$w_\Lambda^{\rm eff}=\frac{B}{(3M_p^2H^2a^3\Omega_\Lambda)^{1+\alpha}}-1\,. \quad (22)$$

By choosing a negative value for B we see that $w_\Lambda^{\rm eff}<-1$, which corresponds to a universe dominated by phantom dark energy. Equation (22), for $\alpha=1$, is the effective parameter of the equation of state for the Chaplygin gas. In this case, in the expression for the energy density (2), the term under the square root should be positive, i.e. $a^6>\frac{-B}{A}$; then the minimal value of the scale factor is given by $a_{\rm min}=(\frac{-B}{A})^{\frac{1}{6}}$, and therefore according to this model we have a bouncing universe. Generally for this model $A>0$, $B<0$ and $1+\alpha>0$. From (15), we can realize that the cosmic scale factor takes values in the interval $a_{\rm min}<a<\infty$, which corresponds to $0<\rho<(2A)^{\frac{1}{1+\alpha}}$, where

$$a_{\rm min}=\left(\frac{-B}{A}\right)^{\frac{1}{3(1+\alpha)}}. \quad (23)$$

Using (2), one can see that the Chaplygin gas model interpolates between dust at small a and a cosmological constant at large a, but choosing a negative value for B, this quartessence idea gets lost. Following [77], if we consider a homogeneous scalar filed $\phi(t)$ and a potential $V(\phi)$ to describe the Chaplygin cosmology, we find

$$\dot{\phi}^2 = \frac{B}{a^6\sqrt{A+\frac{B}{a^6}}}\,. \tag{24}$$

Now, by choosing a negative value for B we see that $\dot{\phi}^2 < 0$, and then we can write

$$\phi = \mathrm{i}\psi\,. \tag{25}$$

In this case the Lagrangian of the scalar field $\phi(t)$ can rewritten as

$$L = \frac{1}{2}\dot{\phi}^2 - V(\phi) = -\frac{1}{2}\dot{\psi}^2 - V(i\psi)\,. \tag{26}$$

The energy density and the pressure corresponding to the scalar field ψ are as follows, respectively:

$$\rho_\psi = -\frac{1}{2}\dot{\psi}^2 + V(\mathrm{i}\psi) \tag{27}$$

$$P_\psi = -\frac{1}{2}\dot{\psi}^2 - V(\mathrm{i}\psi)\,; \tag{28}$$

therefore, the scalar field ψ is a phantom field. This implies that one can generate a phantom-like equation of state from an interacting generalized Chaplygin gas dark energy model in a non-flat universe.

3 Conclusions

In order to solve the cosmological problems and because of our lack of knowledge, for instance, of how to determine what could be the best candidate for DE to explain the accelerated expansion of universe, cosmologists try to approach to results as precise as they can by considering all the possibilities they have. Within the different candidates that may play the role of the dark energy, the Chaplygin gas has emerged as a possible unification of dark matter and dark energy, since its cosmological evolution is similar to an initial dust-like matter model and a cosmological constant for late times. Inspired by the fact that the Chaplygin gas possesses a negative pressure, people [94] have undertaken the simple task of studying the FRW cosmology of a universe filled with this type of fluid.

In this paper, by considering an interaction between the generalized Chaplygin gas energy density and CDM, we have obtained the equation of state for the interacting generalized Chaplygin gas energy density in a non-flat universe. Next we have shown that the interacting generalized Chaplygin gas dark energy with $B < 0$ can be described by a phantom field. Previously I have shown that the phantom dark energy model can behave as a holographic dark energy [95]. On the other hand, recently Zimdhal has shown that a Chaplygin gas can be seen as a special realization of a holographic dark energy cosmology model if the option of an interaction between pressureless dark matter and dark energy is taken seriously [96]. In fact, pressureless dark matter in interaction with holographic dark energy is more than just another model to describe the accelerated expansion of the universe. This may clarify the role of the interaction in this model.

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